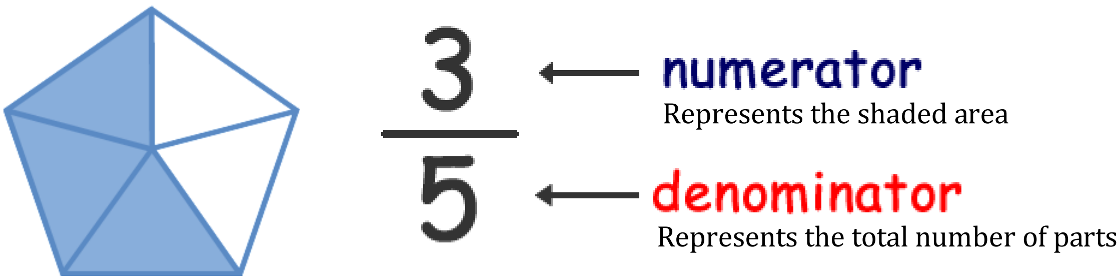
5.1 – Using Models to Add Fractions

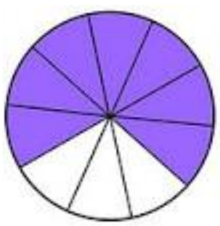
Recall: A fraction is a part of a whole.

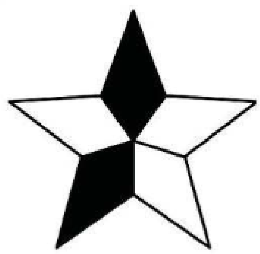
For example, there are 5 equal pieces in the pentagon below. Three pieces are shaded. Therefore, we can say we have 3 out of 5 pieces or

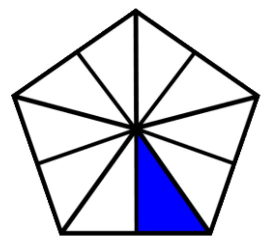


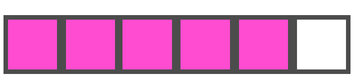
Example 1:

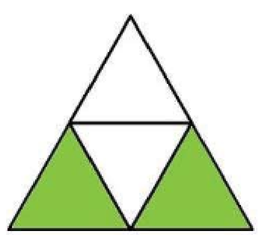
Write the fraction represented by each diagram below.

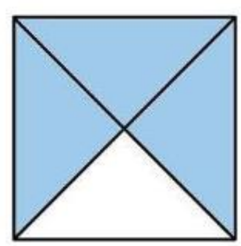
a) 

b) 

c) 

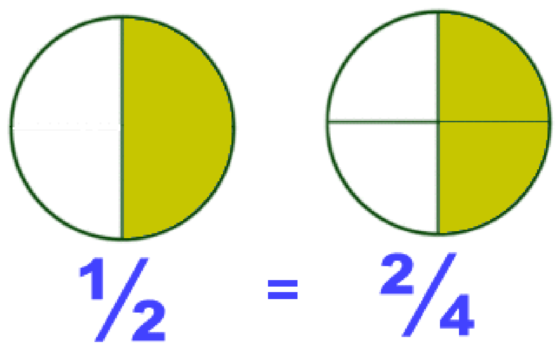
d) 

e) 

f) 

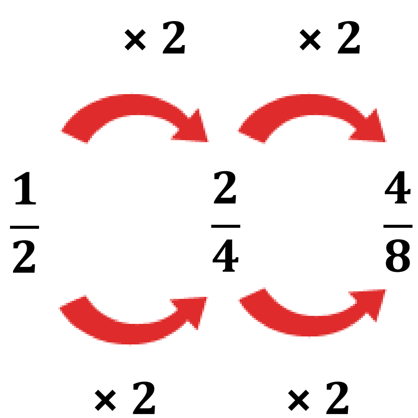
<u>Recall:</u>

Equivalent fractions are two or more fractions of equal value represented by different numerators and denominators.



The above fractions are equivalent fractions. We can easily see that half of the circle is shaded in each diagram, even though the diagrams look a little different.

We can make equivalent fractions by multiplying (or dividing) the numerator and denominator by the same number.



Rule

Change the bottom using multiply or divide …and the same to the top must be applied!

Example 2:

Use fraction circles to represent the following equivalent fractions:

a) $\frac{2}{3} = \frac{4}{6}$

b) $\frac{1}{3} = \frac{4}{12}$

Example 3: Write a fraction equivalent to each of the following:

a) $\frac{3}{4}$

b) $\frac{4}{9}$

c) $\frac{10}{25}$

Example 4: Find the missing number.

a) $\frac{1}{4} = \frac{\_\_}{12}$

b) $\frac{12}{30} = \frac{\_\_}{5}$

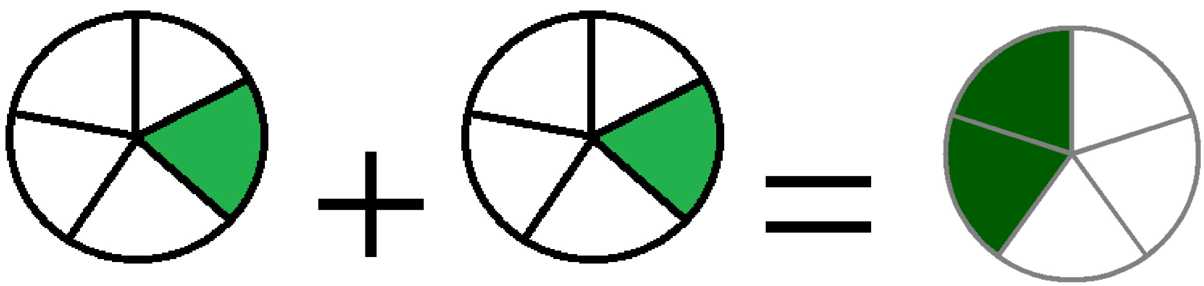
c) $\frac{\_\_}{27} = \frac{2}{9}$

Adding Fractions with Common Denominators

Arlene has two Dairy Milk bars and wants to share them equally among her eight friends. Does it make sense to split one bar in half and split the other bar into the six pieces? Why or why not??

When adding fractions, we can only add if the object is cut into the same number of pieces; that is, if the denominators are the same.

For example:



$\frac{1}{5}$ $\frac{1}{5}$ $\frac{2}{5}$

We can clearly see that one fifth added to one fifth produces two fifths, or $\frac{1}{5}+\frac{1}{5}=\frac{2}{5}$.

Notice that only the numerators are added while the denominators are kept the same.

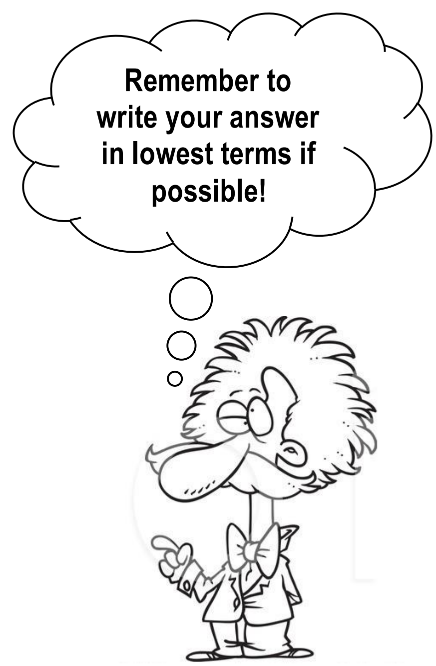
Example 1: Add using a model.

a) $\frac{1}{4} + \frac{3}{4}$

b) $\frac{2}{15} + \frac{8}{15}$

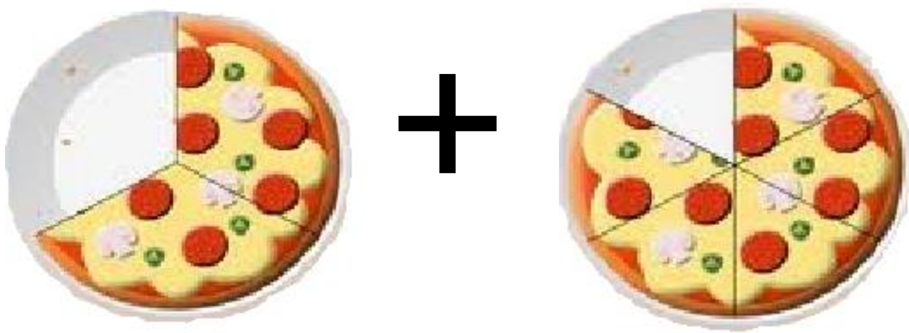
c) $\frac{2}{7} + \frac{3}{7}$

d) $\frac{1}{9} + \frac{2}{9}$



Adding Fractions with Different Denominators

Sometimes we will need to add fractions that don't have a common denominator. Look at the pizzas below…we can't simply add the pieces in each pizza (numerators) because the pieces are not the same size.



$\frac{2}{3}$ $\frac{5}{6}$

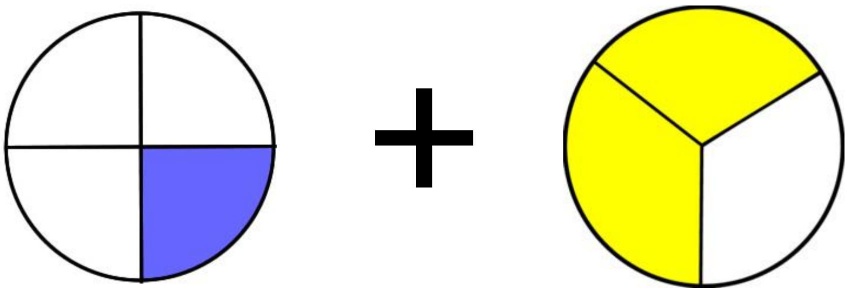
In a case such as this, we need to cut the pieces so we have the same size pieces in each pizza.

We have $\frac{4}{6}$ and $\frac{5}{6}$

These pieces can now be added because they are the same size (the denominators are the same).

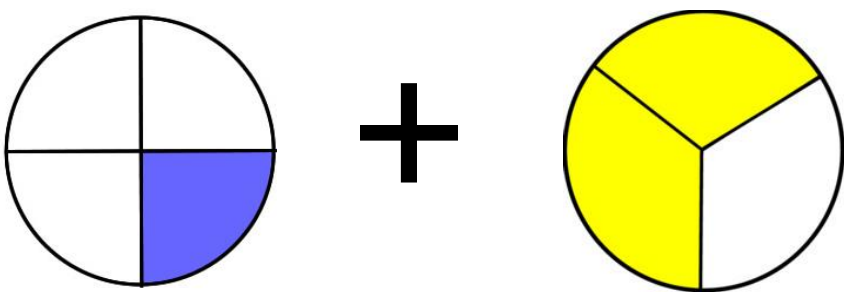
Thus, $\frac{4}{6}+\frac{5}{6}=\frac{9}{6}$ or $1\frac{3}{6}$ which can be reduced to $1\,\frac{1}{2}$.

Let's try adding $\frac{2}{3} + \frac{1}{4}$



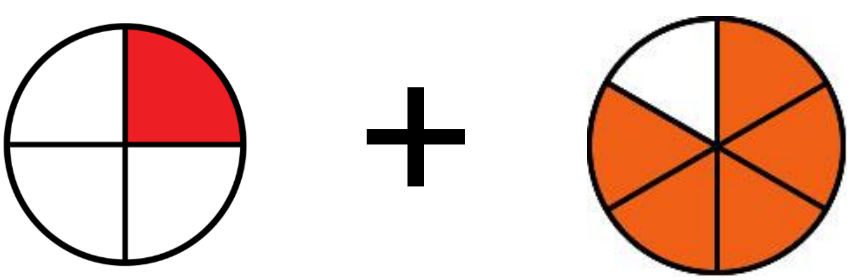
What do we need to do in this case to make sure both pizzas have the same number of pieces? What is the smallest number of pieces we must cut each pizza into, to have the same number of pieces in each?

Let's try cutting them!



We now have: ________________ + ________________ = ____________________

Let's try another example:



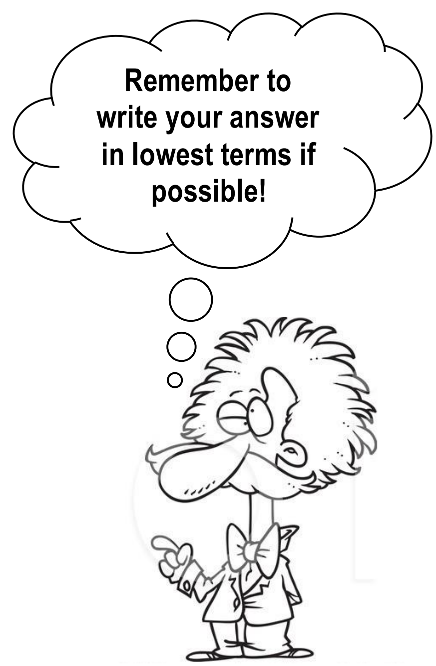
Example 1: Use a model to add the following.

a) $\frac{1}{2}+\frac{3}{4}$

e) $\frac{2}{3}+\frac{2}{9}$

f) $\frac{2}{6}+\frac{11}{12}$

g) $\frac{1}{3}+\frac{1}{2}$

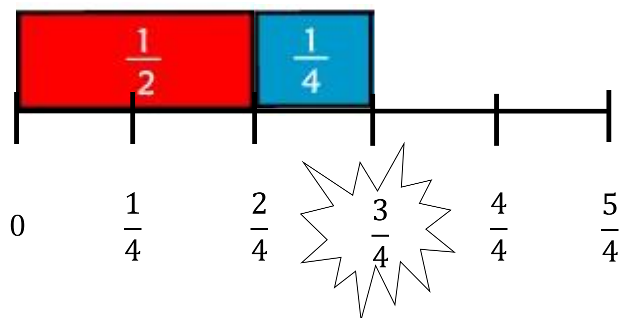


5.2 – Other Models for Adding Fractions

We can also use fraction strips and number lines to add fractions.

To add $\frac{1}{2}+\frac{1}{4}$, we will need a strip that represents $\frac{1}{2}$ and another that represents $\frac{1}{4}$.

Since the common denominator is 4, we cut our number line into fourths and place the strips on the number line:



Therefore, we can easily see $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$.

Let’s try another example: $\frac{1}{2}+\frac{2}{3}$

Example 1: Add using fraction strips.

a) $\frac{1}{3} + \frac{1}{2}$

b) $\frac{1}{4} + \frac{1}{6}$

c) $\frac{2}{3} + \frac{1}{5}$

d) $\frac{5}{6} + \frac{1}{3}$

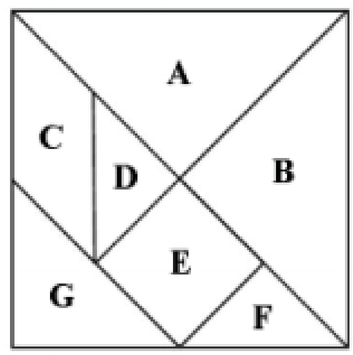
Example 2: Is each sum greater than 1 or less than 1? How can you tell?

a) $\frac{2}{6}+\frac{1}{6}$

b) $\frac{7}{10}+\frac{4}{10}$

Example 3:

A tangram is a square puzzle that is divided into seven shapes.



Given that piece A is $\frac{1}{4}$ of the whole square:

a) What are the values of pieces B, C, D, E, F and G?

B _______ C _______ D _______

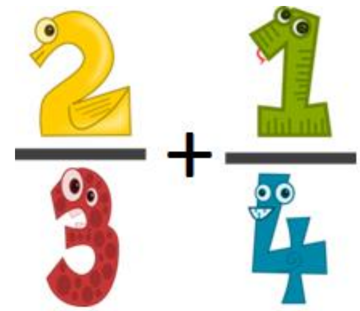
E _______ F _______ G ________

b) What is the sum of A and B? B and G? E and F?

A + B __________ B + G __________ E + F __________

c) Which two tangram pieces add up to the value of B? C?

5.3 – Using Symbols to Add Fractions



In the last section, we learned that when adding fractions, we need to make sure we have the same denominator first; that is, we need to make sure our pizzas are cut into the same size pieces.

If the denominators are already the same, add the numerators. Don't change the denominator. Write your answer in lowest terms where possible.

Example 1: Add the following without using a model.

a) $\frac{5}{9}+\frac{2}{9}$

b) $\frac{1}{7}+\frac{6}{7}$

c) $\frac{5}{11}+\frac{8}{11}$

d) $\frac{7}{16}+\frac{3}{16}$

e) $\frac{2}{15}+\frac{7}{15}$

f) $\frac{12}{13}+\frac{15}{13}$

g) $\frac{3}{8}+\frac{5}{8}$

h) $\frac{11}{28}+\frac{7}{28}$

g) $\frac{19}{24}+\frac{7}{24}$

To add fractions with different denominators, we need to find equivalent fractions with a **common denominator**. That means, we need to find the smallest number that both denominators will go into equally so that both pizzas will have the same number of parts.

Least Common Denominator

For example: $\frac{1}{2} + \frac{3}{5}$

Step 1: Find a common denominator.

What is the smallest number that both 2 and 5 go into evenly?

Step 2: We now need write equivalent fractions for each fraction with the LCD (lowest common denominator).

Step 3: Add the new numerators only!

Example 1:

a) $\frac{1}{2}+\frac{1}{10}$

b) $\frac{2}{3}+\frac{5}{6}$

c) $\frac{5}{9}+\frac{1}{3}$

d) $\frac{1}{2}+\frac{2}{5}$

e) $\frac{1}{4}+\frac{2}{5}$

f) $\frac{4}{7}+\frac{1}{5}$

g) $\frac{1}{9}+\frac{6}{7}$

h) $\frac{11}{14}+\frac{2}{7}$

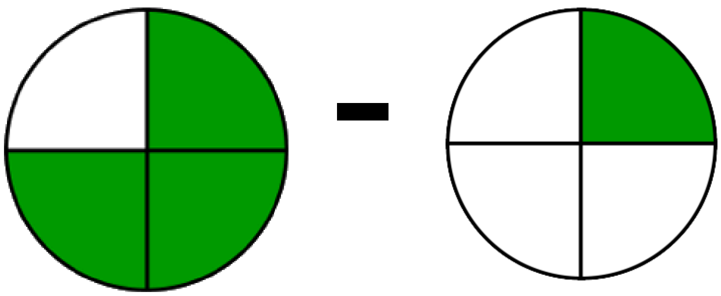
i) $\frac{3}{5}+\frac{1}{4}$

5.4 – Using Models to Subtract Fractions

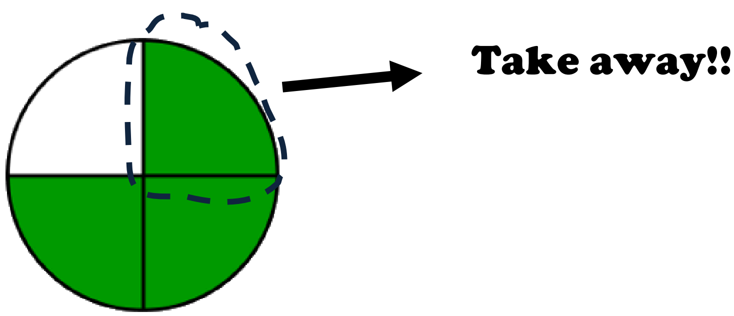
We subtract fractions the same way we add fractions, that is, we have to make sure our pieces are the same size.

If our denominators are the same, we simply subtract the numerators.

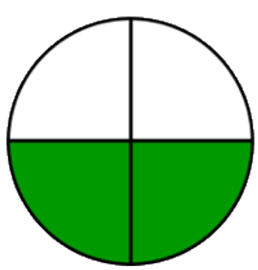
For example: $\frac{3}{4} - \frac{1}{4}$



The diagram indicates that we need to take $\frac{1}{4}$ away from $\frac{3}{4}$.



Which leaves us with



So, we can easily see that $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$

We can easily see that like with addition, the denominators stay the same. Only the numerators are subtracted.

Example 1: Subtract the following using a model.

a) $\frac{5}{6} - \frac{3}{6}$

b) $\frac{7}{8} - \frac{3}{8}$

c) $\frac{4}{5} - \frac{1}{5}$

d) $1 - \frac{4}{7}$

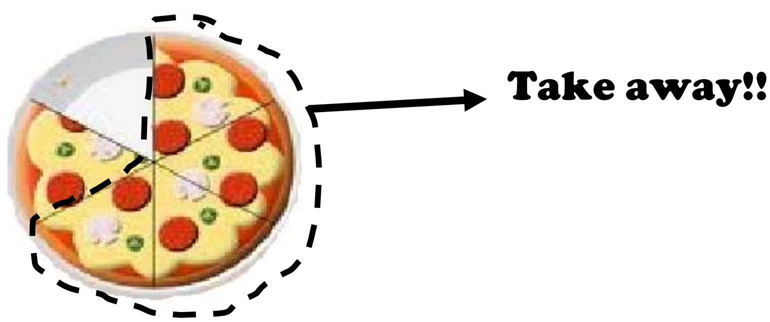
Like when adding fractions, we need to make sure our denominators are the same; that is, we need to cut our pizzas into the same size pieces.

$\frac{5}{6}$ $\frac{2}{3}$

In this case, we need to cut the pieces so we have the same size pieces in each pizza.

Now we have $\frac{5}{6}$ and $\frac{4}{6}$

These pieces can now be subtracted because they are the same size (the denominators are the same).



We are left with one piece of pizza.

Thus, $\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

Example 2: Subtract the following by using a model.

a) $\frac{1}{2} - \frac{1}{10}$

b) $\frac{2}{3} - \frac{1}{2}$

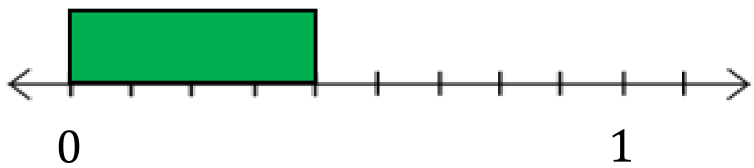
c) $\frac{5}{9} - \frac{1}{3}$

We can also use fraction strips to subtract fractions.

For example: $\frac{4}{9} - \frac{1}{3}$

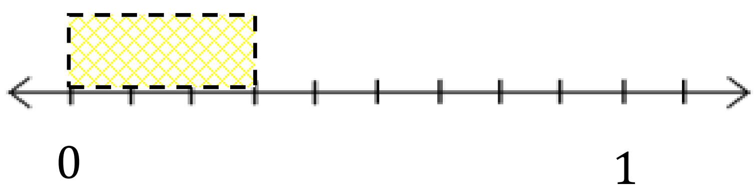
First, draw a number line divided into NINTHS since 9 is the lowest common denominator.

Shade $\frac{4}{9}$

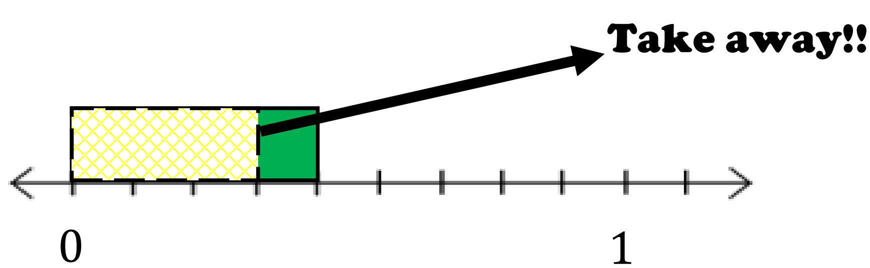


$\frac{1}{3}$ is the same as $\frac{3}{9}$, which we need since our line is divided into ninths.

This can be represented as follows.



We can now remove the $\frac{1}{3}$ or $\frac{3}{9}$ from the $\frac{4}{9}$



We can see that there is $\frac{1}{9}$ remaining. So our final equation is $\frac{4}{9} - \frac{1}{3} = \frac{1}{9}$

Example 3: Subtract using fraction strips.

a) $\frac{5}{8} - \frac{1}{2}$

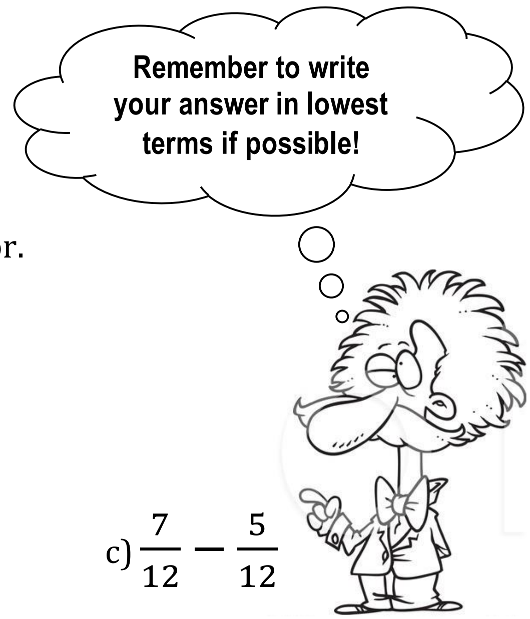
b) $\frac{3}{6} - \frac{1}{3}$

c) $\frac{6}{10} - \frac{2}{5}$

d) $\frac{11}{12} - \frac{1}{3}$

5.5 – Using Symbols to Subtract Fractions

To subtract fractions without models, we first make sure the denominators are the same. If they are the same, we simply subtract the numerators. If the denominators are not the same, we find the lowest common denominator and rewrite the fractions, then subtract the numerators.



Remember:

Step 1: Find a common denominator (if needed).

Step 2: Re-write each fraction with the common denominator.

Step 3: Subtract the new numerators only!!

Example 1: Subtract the following without models.

a) $\frac{4}{5} - \frac{1}{5}$

b) $\frac{5}{9} - \frac{4}{9}$

c) $\frac{7}{12} - \frac{5}{12}$

d) $\frac{1}{2} - \frac{1}{4}$

e) $\frac{5}{6} - \frac{2}{3}$

f) $\frac{5}{9} - \frac{1}{3}$

g) $\frac{1}{2} - \frac{2}{5}$

g) $\frac{3}{4} - \frac{2}{5}$

i) $\frac{4}{7} - \frac{1}{5}$

j) $\frac{8}{9} - \frac{6}{7}$

k) $\frac{11}{14} - \frac{2}{7}$

l) $\frac{3}{5} - \frac{1}{4}$

Example 2:

Glenn has $\frac{5}{8}$ of a cup of walnuts. He needs $\frac{2}{3}$ of a cup of walnuts to make a loaf of banana bread. Does Glenn have enough? If your answer is yes, explain why it is enough.
If your answer is no, how much more does Glenn need?

Example 3:

Brandy spent $\frac{1}{10}$ of her summer vacation reading, $\frac{1}{15}$ watching her favorite movies, $\frac{1}{3}$ visiting her grandparents, and twice the reading time playing with her friends.

a) What is the difference in the fractions Brandy spent with her grandparents and playing with her friends?

b) Did she spend more time reading or watching movies? Explain.

c) Did Brandy have time to do anything else besides these activities? Explain.

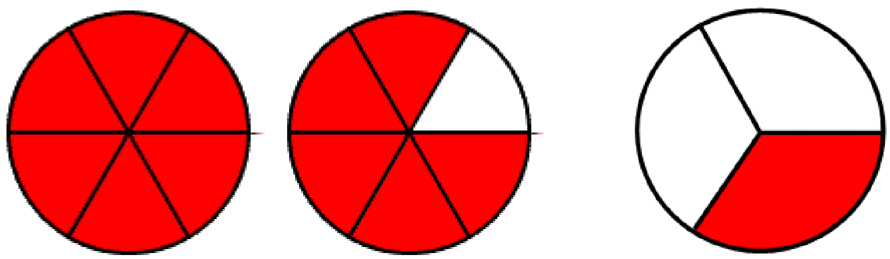
Example 4: When one fraction is subtracted from another fraction, the difference is zero. The fractions have different denominators. What might the fractions could be? Give two possible answers.

5.6 – Adding with Mixed Numbers

When adding mixed number it is always important to first estimate the sum to make sure your answer is reasonable.

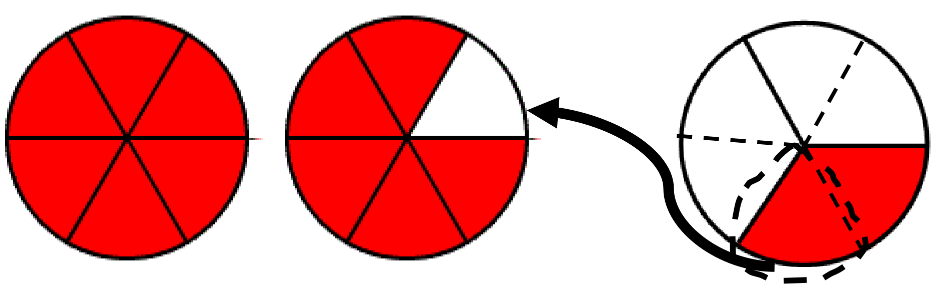
For example, $1\frac{5}{6}+\frac{1}{3} \rightarrow 1\frac{5}{6}$ is a little less than 2 and $\frac{1}{3}$ is a little less than a half so the sum should be a bit more than 2.

To find the actual sum we can model each fraction.

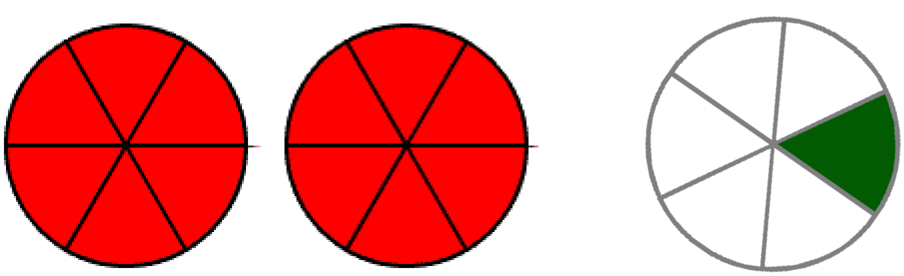


$$1\frac{5}{6} \qquad + \qquad \frac{1}{3}$$

Split the three pieces into six pieces. Use one of the six pieces to fill in the whole pizza.



We can easily see that $1\frac{5}{6}+\frac{1}{3}=2\frac{1}{6}$



The best way to add mixed numbers is to change them to improper fractions, and then use the rules we have already learned for adding:

- We must have COMMON DENOMINATORS to add fractions!!
- Only add the numerators leaving the denominator the same.

For example:

$1\frac{1}{2} + 2\frac{3}{4}$

Example 1:

a) $1\frac{4}{5} + 2\frac{1}{2}$

b) $2\frac{1}{3} + 1\frac{1}{6}$

c) $5\frac{3}{5} + 2\frac{4}{5}$

5.7 – Subtracting with Mixed Numbers

When we subtract mixed numbers, we do the same as adding them – change them to improper fractions, and then use the rules we have already learned for subtracting:

- Find a COMMON DENOMINATOR and change the fractions to equivalent fractions.
- Only subtract the numerators leaving the denominator the same.

For example:

$3\frac{1}{2} - 2\frac{3}{4}$

Example 1:

a) $2\frac{4}{5} - 2\frac{1}{2}$

b) $2\frac{1}{12} - 1\frac{1}{6}$

c) $5\frac{3}{5} - 2\frac{4}{5}$

d) $2 - 1\frac{1}{6}$