Section 6.1 – Solving Equations

An **algebraic expression** is a mathematical sentence with numbers and/or variables connected by $+, -, \times, \div$.

There is <u>NO</u> equals sign.

Let's consider: $2p - 7$

- p is the variable
- 2 is the numerical coefficient
- -7 is the constant term

An **equation** is a statement that shows two expressions are equal.

It <u>MUST</u> have an equals sign.

For example,

$5x$ and $-6x + 3$ are **expressions.**

$5x = 15$ and $-6x + 3 = 57$ are **equations.**

<u>Example 1:</u> Classify each of the following as an equation or an expression.

a) $4c + 9$ _____

b) $\frac{2x}{5} = 6$ _____

c) $3x = 18$ _____

d) $5n + 8$ _____

e) $7x + 4$ _____

f) $8q$ _____

g) $2t - 6 = 28$ _____

h) $v - 2 = 12$ _____

Language of Mathematics

Addition	Subtraction	Multiplication	Division	Equals
•sum •plus •add •and •more than •total (of) •increased by •raise •combined •in all •altogether •additional •together •both •added to	•less (than) •minus •decrease •difference •reduce •lost left •remaining •fell •dropped •change •farther •diminished •how much more •how much less •subtract from	•multiplied •times •total •of •per •as much •twice •by •area •product •apiece •doubled •tripled	•divided (evenly) •quotient •split •each •cut •equal pieces •average •every •out of •shared •how many times •shared equally •per •part	•is/are •the same as •equals •equal to •result is

Example 2: Write an equation for each sentence.

A) A number added to twelve is twenty-five. _____

B) The product of a number and seven equals forty-two. _____

C) Seven more than twice a number is twenty-three. _____

D) A number divided by five and added to seven is thirteen. _____

E) The difference between triple a number and six is fifteen. _____

F) Double a number subtract nine results forty-three. _____

There are several different ways to solve an equation. Some of these include:

1. Systematic Trial
2. Inspection
3. Models
4. Algebra

Method 1: Systematic Trial

This means choosing a value for the variable and plugging it into the equation to get the answer. Otherwise known as "Guess and Check" or "Trial and Error".

Let's consider the equation: $\boldsymbol{3d + 2 = 35}$

Try $d = 2$ Try $d = 7$ Try $d = 11$

Method 2: Inspection

This means looking closely at the equation and carefully figuring out the value of the variable.

Let's consider the equation $2n - 5 = 25$

Cover up the $2n$.  $- 5 = 25$

Now think: WHAT $- 5 = 25$

_____ $- 5 = 25$

We can easily see that $2n =$ _____, so $n =$ _____.

Example 3: Solve using systematic trial: $4d + 5 = 33$

Example 4: Solve using inspection: $6m - 4 = 50$

Example 5: Solve using the method of your choice.

a) $2n + 3 = 9$

b) $x + 15 = 21$

c) $\frac{t}{5} + 2 = 8$

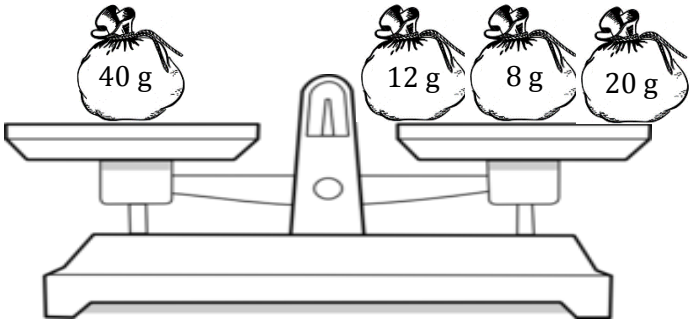
d) $2p - 6 = 10$

Section 6.2 – Using a Model to Solve Equations

We can also use a **balance** to model an equation.

When both sides of the balance are at the same height, they are equal.

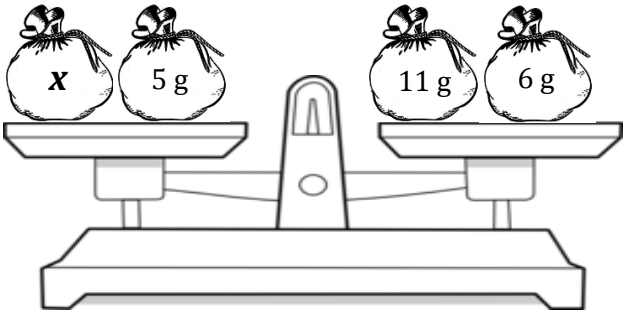
For example:



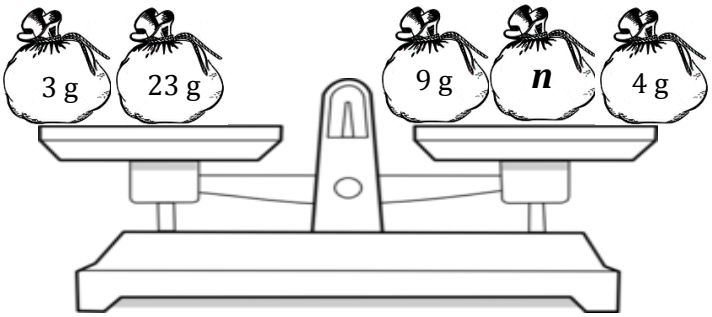
How would we represent this situation as an equation?

Example 1: Write an equation for each situation below and determine the missing value.

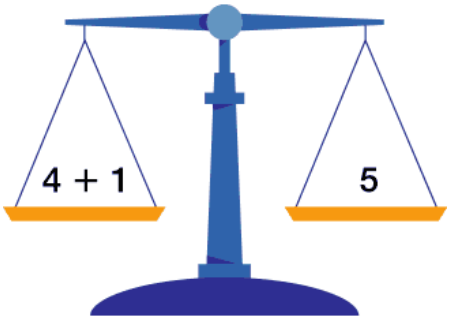
A)



B)



When using balances, the key is to make sure that the left side of the balance is equal to the right side of the balance.



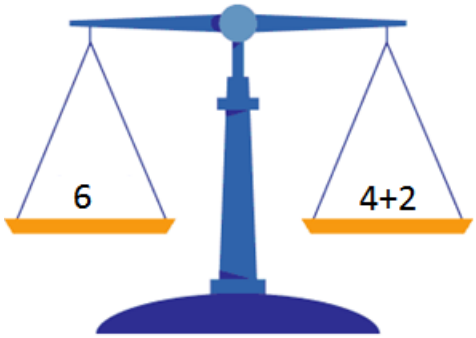
This equation is balanced – that it, the left side is equal to the right side.

Left Side = Right Side

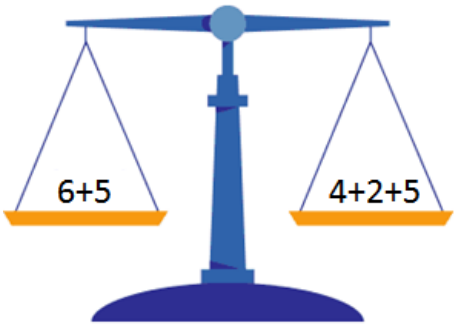
$4 + 1 = 5$

$5 = 5$

Consider the following. Is this balanced?



What happens if we add 5 to both sides of the balance?



Since we must make sure we keep the left side equal to the right side, we can easily see that if we add (or subtract) something to one side, we **MUST** do the same thing to the other side to keep the balance.

Let's consider the following balance. Write the equation.

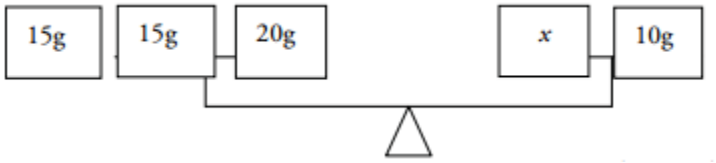
What could we do to get the value of x?

Since we must keep the equation balanced, if we remove 3 from one side we **must** remove 3 from the other side too.

We can easily see that $x =$ _____.

Example 2: Find the value of the unknown mass and sketch the steps used.

a)

b) 

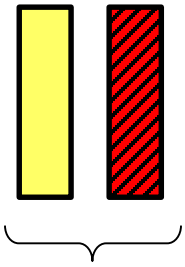
Example 3: Sketch balance scales to represent each equation. Solve and verify the solution.

a) $2y = 18$

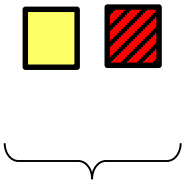
b) $3n + 2 = 17$

Section 6.3 – Solving Equations Involving Integers

Recall algebra tiles from unit 2:



These are *x* tiles.
They represent a variable.



These are unit tiles.
They represent constants.

Remember! Yellow tiles (unshaded) are positive. Red tiles (shaded) are negative.
One positive and one negative tile together form a zero pair.

Example 1: Represent the following equations using algebra tiles.

a) $x + 4 = 9$

b) $2x - 3 = 15$

c) $4x - 1 = 11$

Example 2: Draw the algebra tiles for each expression below, then draw the opposite of each.

a) $+2$

b) $-4x$

c) -7

d) $3x$

Example 3: Write the equation for each statement. Draw the tiles for each equation.

a) Three more than double a number is thirteen.

b) Six more than a number is fourteen.

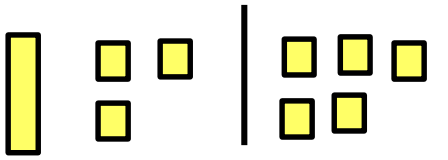
c) Triple a number decreased by two is ten.

d) Twice a number increased by three is five.

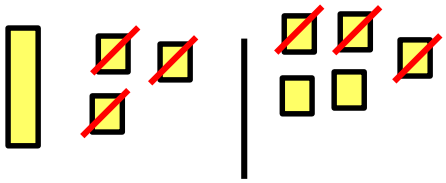
e) Fourteen decreased by twice a number equals eight.

We can use algebra tiles to solve equations as well. Remember, one red and one yellow tile cancel each other out. We can use this idea to isolate the variable.

Let's consider the equation: $x + 3 = 5$

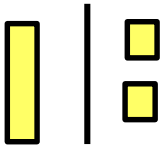


$$x + 3 = 5$$



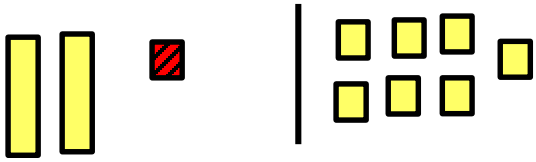
Subtract three from each side

$$x + 3 - 3 = 5 - 3$$

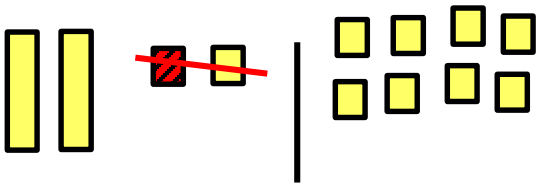


$$x = 2$$

Let's consider another equation: $2x - 1 = 7$

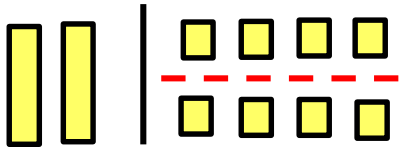


$$2x - 1 = 7$$



Add one positive tile to cancel the negative on the left – we must add the same to the right side. Simplify.

$$2x - 1 + 1 = 7 + 1$$
$$2x = 8$$



Since we have two *x* tiles, we split the right side into two groups. Each *x* tile is paired with 4 unit tiles.

$$x = 4$$

Example 4:

Draw the algebra tiles for each equation below.
Solve each equation showing all steps. Verify your answer.

a) $n - 3 = 4$

b) $h + 1 = -2$

c) $2 = y - 6$

d) $w - 4 = 1$

e) $2x - 1 = 9$

Example 5: Show whether or not $x = 7$ is the solution to each of the following equations:

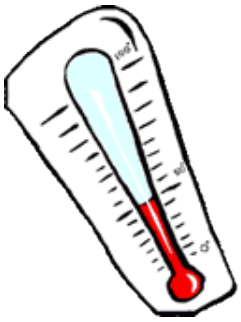
a) $6x = 48$

b) $3x + 2 = 20$

c) $\frac{x}{7} = 1$

Example 6: Write an equation for each problem. Use algebra tiles to solve and verify.

a) The temperature dropped 5 ℃ to -2 ℃. What was the original temperature?

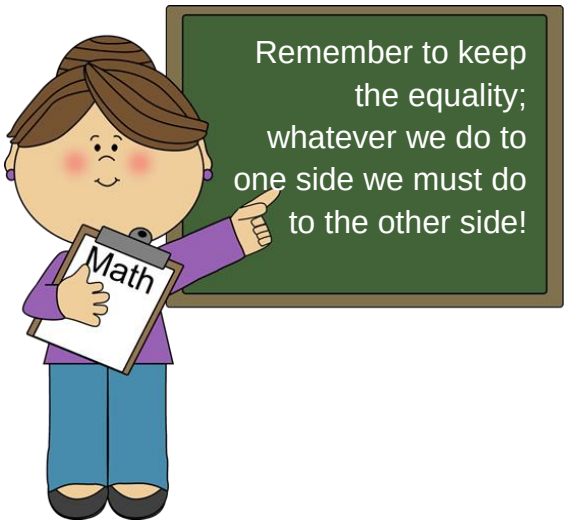


b) Frank is 9 years old. He is 4 years older than Joe. How old is Joe?

c) Susan borrowed books from the library. She then returned 4 books. If she still has 3 books at home, how many did she borrow?

Section 6.4 – Solving Equations Using Algebra

Solving equations using algebra will allow us to solve problems with larger numbers.
We can recall the steps we used with models such as a balance scale or algebra tiles.



When solving an equation, our goal is to get the variable on one side of the equation by itself and everything else on the other side.

Example 1: Solve the following problems using algebra and verify by substitution.

a) $x + 4 = 7$

b) $x - 1 = 5$

c) $3x = 24$

d) $\frac{x}{7} = 7$

e) $2x + 3 = 9$

f) $6x + 4 = 29$

g) $2x + 1 = 13$

Example 2:

There are 38 boys. This is 6 more than double the number of girls. Write an equation and solve using algebra.

Example 3:

The cost of a meal is shared by 5 people and is equal to $35 each. Write an equation and solve using algebra.

Example 4:

60 centimeters is one half of Bob's height.
Write an equation, solve and verify.

Example 5:

Derek ate one third of a pizza. This was 2 slices. How many slices of pizza were in the whole pizza? Write the equation, solve and verify.

Section 6.5 – Using Different Methods to Solve Equations

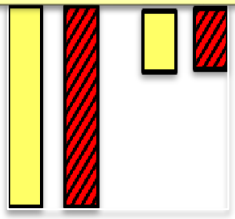
We've looked at several ways to solve equations:

Systematic Trial

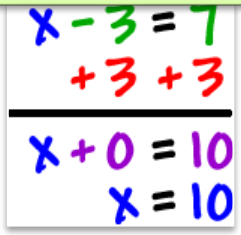
Inspection

Balance Scales

Algebra Tiles



Algebra



Example 1: Use any of the above listed methods to solve the following equations.

a) $n + 3 = 12$ b) $x + 8 = 19$ c) $f - 2 = 11$

d) $d - 5 = 23$ e) $m + 11 = 21$ f) $3p = 12$

g) $7h = 56$ h) $2h + 7 = 23$ i) $5t - 7 = 28$

j) $4k - 9 = 15$ k) $7m - 22 = 18$ l) $2b + 3 = 7$

Example 2: Twelve years ago, Mary was 13 years old. How old is she now? Write an equation, solve and verify.

Example 3:

$\frac{1}{5}$ of a chocolate bar weighs 9g. What is the weight of the total bar?
Write an equation, solve and verify.

Example 4:

Susan baked some cookies and shared then equally among three of her students. If each student received five cookies, how many cookies did Susan bake?

Write the equation, solve and verify.