

Mathematics 7

Fractions, Decimals and Percentages

FRACTIONS:

- $\frac{50}{100}$ **Numerator** (*top number*)
 Denominator (*bottom number*)

* means $50 \div 100$

There are three types of fractions:

1.) *Proper Fraction*

▶ $\frac{13}{25}$

The denominator is greater than the numerator

2.) *Improper Fractions*

▶ $\frac{30}{10}$ or $\frac{18}{18}$

The denominator is less than or equal to the numerator

3.) *Mixed Fractions*

▶ $3 \frac{8}{10}$

The fraction has a whole number in front of the fraction

* A whole number (0, 1, 2, 3, 4, 5, 6, 7, 8, ...) can be written as a fraction .

▶ $24 = \frac{24}{1}$

Finding equivalent Fractions:

- Equivalent fractions mean that two fractions have the same value but look different

Example #1: *Simply Add Zero(s)*

$$\frac{2}{8} = \frac{20}{80} = \frac{200}{800}$$

Example #2: *Multiply the fraction's numerator and denominator by the same number*

$$\frac{2}{8} = \overset{x \ 2}{\frac{4}{16}} = \overset{x \ 2}{\frac{8}{32}}$$

Example #3: *Divide the fraction's numerator and denominator by the same number*

$$\frac{2}{8} = \overset{\div \ 2}{\frac{1}{4}}$$

Converting Fractions to Decimals:

- Example #1 - Proper Fractions: *Use long-division.*

$$\frac{2}{7} = \frac{0.2828}{7 \overline{)2.0}} = \overline{0.28}$$

$$\begin{array}{r} 7 \overline{)2.0} \\ \underline{-14} \\ 60 \\ \underline{-58} \\ 20 \\ \underline{-14} \\ 60 \end{array}$$

And so on

- Example #2 - Improper Fractions: *Convert to a Mixed fraction to determine the whole number, then use long-division to convert the fraction portion to a decimal.*

Step 1 $\frac{11}{5} = 11 \div 5 = 2 \frac{1}{5}$ The whole number is **2**

Step 2 $\frac{1}{5} = \frac{0.2}{5 \overline{)1.0}} = 0.2$

$$\begin{array}{r} 5 \overline{)1.0} \\ \underline{-10} \\ 0 \end{array}$$

0 Remainder

Put the whole number and the decimal together.

$$\frac{11}{5} = 2.2$$

- Example #3 - Mixed Fraction: *Converting mixed fractions to decimal is exactly like converting improper fractions to decimal minus step 2 because it is already a mixed fraction (See example 2).*

Determining Which Fraction is Greater:

$$\frac{2}{7} \quad \text{or} \quad \frac{3}{8}$$

Hint: convert the fraction to decimal using Long-Division, Then use a number line or place value chart. Of course, mental math is the preferred method.

$$\overline{0.28} \quad \text{or} \quad 0.375$$

0.375 is greater than $\overline{0.28}$. The three in the tenths place of 0.375 is greater than the 2 in the tenths place of 0.28.

Subtracting Decimals:

$$\begin{array}{r} 123.67 \\ - 42.89 \\ \hline 80.78 \end{array}$$

Align the decimals and subtract accordingly to find the *difference*.
Drop the decimal immediately below.

$$\begin{array}{r} 675.9 \\ - 22.74 \\ \hline 653.16 \end{array} \quad \leftarrow \text{Difference}$$

Multiplying Decimals:

Step 1: Understand the operation and decimal numbers.

$$2.3 \times 5.8$$

Step 2: Align the numbers underneath each other in the order they appear linearly

$$\begin{array}{r} 2.3 \\ \times 5.8 \\ \hline \end{array}$$

Step 3: Ignore the decimals *for now* and multiply as you would normally.

$$\begin{array}{r} 23 \\ \times 58 \\ \hline 184 \\ + 1150 \\ \hline 1334 \end{array}$$

Step 4: count and add the number of places the digit to the extreme right of the decimal is for each number being multiplied in step 1, then place the decimal the sum of the count from the extreme right of the *product*.

$$13.34 \quad \leftarrow \text{Product}$$

$$2.3 \times 5.8 = 13.34$$

Dividing Decimals:

Step 1: Understand the operation and decimal numbers.

$$54.6 \div 4.6$$

Step 2: Align the numbers as for long-division, making sure to move the decimal to the extreme right until it is no longer in the number. Move the decimal in the dividend an equal amount of spaces to the right.

$$46 \overline{)546.}$$

Step 3: Divide as you would normally for long-division.

$$\begin{array}{r}
 \underline{11.86} \quad \leftarrow \textit{Quotient} \\
 \textit{Divisor} \rightarrow 46 \overline{)546.} \quad \leftarrow \textit{Dividend} \\
 \underline{-46} \\
 86 \\
 \underline{-46} \\
 400 \\
 \underline{-368} \\
 320 \\
 \underline{-276} \\
 44 \text{ Remainder}
 \end{array}$$

Step 4: Place the decimal directly above in the quotient.

$$\begin{array}{r}
 \underline{\hspace{10em}} 11.86 \leftarrow \textit{Quotient} \\
 \underline{\hspace{10em}} 54.6 \div 4.6 = 11.86
 \end{array}$$

Another Example:

$$\underline{\hspace{1em}} 2.24 \div 0.7$$

$$\begin{array}{r}
 \underline{3.2} \\
 7 \overline{)22.4} \\
 \underline{-21} \\
 14 \\
 \underline{-14} \\
 0 \text{ Remainder}
 \end{array}$$

$$2.24 \div 0.7 = 3.2$$

ORDER OF OPERATIONS:

Step 1: Do the operations in brackets first

Step 2: Divide and multiply, in order from left to right

Step 3: Add and subtract, in order from left to right

Example:

$$0.39 + 16.3 \times (2.3 + 4.8) - 22 \div 2$$

Do the operations in brackets first

$$2.3 + 4.8 = 7.1$$

$$0.39 + 16.3 \times 7.1 - 22 \div 2$$

Divide and multiply, in order from left to right

$$16 \times 7.1 = 115.73$$

$$22 \div 2 = 11$$

$$0.39 + 115.73 - 11$$

$$0.39 + 115.73 = 116.12$$

Add and subtract, in order from left to right

$$116.12 - 11 = 105.12$$

$$105.12$$

PERCENTAGES:

- ▶ 52% is an example of a percentage
A percentage is a number out of one hundred

$$\frac{52}{100}$$

$$52\% \times .01 = 0.52$$

Converting a decimal to a percentage:

- Take the decimal and multiply by 100, then add the % (percent) sign

$$0.52 \times 100 = 52$$

$$0.876 \times 100 = 87.6\%$$

Converting a percentage to a decimal:

- Take the percentage and divide by 100

$$64\% \div 100 = 0.64$$

$$3\% \div 100 = 0.03$$

Converting a fraction to a percentage:

Step 1: Divide the **numerator** (*top number*) of the fraction by the **denominator** (*bottom number*) of the fraction.

$$\frac{89}{95} \qquad 89 \div 95 = 0.936$$

Step 2: Take the quotient and multiple by 100, then add the % (percent) sign

$$0.936 \times 100 = 93.6\%$$

Calculating a percentage of a number:

- Convert the percentage to a decimal, then multiply the decimal and the number.

$$5\% \text{ of } 86 =$$

$$5\% = 5\% \div 100 = 0.05$$

$$86 \times 0.05 = 4.3$$

Sales Tax:

- In order for the government to pay for the expenses (military, health care, education, road construction, etc.) of running and maintaining a country at the federal and provincial level, it raises money through a sales tax. A sales tax is a fee put on every item sold. The HST (Harmonized Sales Tax) is a combination of two sales taxes; the GST (General Sales Tax) at the federal level and the PST (Provincial Sales Tax) at the provincial level. The HST is currently 13%.

Word Problem

John paid \$18.50 for a new tempered saw. The HST was 22%. How much did John pay in total?

Step 1: Convert the percent to a decimal.

$$22 \div 100 = 0.22$$

Step 2: Multiple the decimal of the convert percent in to the cost of the item.

$$\begin{array}{r} 18.50 \quad \leftarrow \quad 2 \text{ decimal places} \\ \times \quad \underline{0.22} \quad \leftarrow \quad + \quad \underline{2 \text{ decimal places}} \\ \hline 3700 \quad \quad \quad 4 \text{ decimal places} * \\ + \quad \underline{37000} \\ \hline 4.0700 \end{array}$$

Place the decimal four places to the left
Round to the nearest cent; 4.07

$$\text{HST} = \$4.07$$

Step 3: *Add* the HST to the cost of the item

$$\begin{array}{r} \$18.50 \quad * \text{ Line up the decimals} \\ + \quad \underline{\$4.07} \\ \hline \$22.57 \end{array}$$

Step 4: Write a statement

John paid a total of \$22.57 for the tempered saw at the general store.

Discounts:

- Sally wanted to buy a pair of sneakers at the general store. The advertised sale on the tag was \$75.99. The sneakers had a discounted price of 30%. If the sales tax was 13%. How much did Sally pay for the sneakers?

- Calculate the discount

Step 1: Convert the discounted percent to a decimal.

$$30 \div 100 = 0.3$$

Step 2: Multiple the decimal of the convert percent in to the cost of the item.

$$\begin{array}{r} 75.99 \quad \leftarrow \quad 2 \text{ decimal places} \\ \times \quad \underline{0.3} \quad \leftarrow \quad + \quad \underline{1 \text{ decimal places}} \\ \hline 227970 \quad \quad \quad 3 \text{ decimal places} * \end{array}$$

Place the decimal three places to the left
Round to the nearest cent; 22.80

$$\text{Discount} = \$22.80$$

Step 3: *Subtract* the discount from the cost of the item

$$\begin{array}{r} \$75.99 \quad * \text{ Line up the decimals} \\ - \quad \underline{\$22.80} \\ \hline \$53.19 \end{array}$$

Step 4: Write a statement

The price of the sneakers with the discount taken off, without the sales tax added is \$53.19.

- Step Two - Calculate the sales tax

Step 1: Convert the sales tax percent to a decimal.

$$13 \div 100 = 0.13$$

Step 2: Multiple the decimal of the convert percent in to the cost of the item.

$$\begin{array}{r} 53.19 \quad \leftarrow \quad 2 \text{ decimal places} \\ \times \quad \underline{0.13} \quad \leftarrow \quad + \quad \underline{2 \text{ decimal places}} \\ \hline 15957 \quad \quad \quad 4 \text{ decimal places} * \\ + \quad \underline{53190} \\ \hline 6.9147 \end{array}$$

Place the decimal four places to the left
Round to the nearest cent; 6.91

$$\text{Sales tax} = \$6.91$$

Step 3: *Add* the sales tax to the cost of the item

$$\begin{array}{r} \$53.19 \\ + \$ \underline{6.91} \\ \$60.10 \end{array} \quad * \text{ Line up the decimals}$$

Step 4: Write a statement

The price of the sneakers with the discount taken off and the sales tax added is 60.10. Sally paid \$60.10 for the pair of sneakers.